**Does Constraining the Search Space of GA Always Help? The Case of Balanced Crossover Operators**

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**Problem**

- In several combinatorial optimization problems, the feasible solutions are represented by balanced binary strings, composed of an equal number of zeros and ones.
- Classic crossover operators in GA (e.g., one-point) do not preserve balancedness in general:

```plaintext
p1: 1|1|0|1|0|1|0|0
→ \(#0 = 4, #1 = 5\)
\(\chi\) point

p2: 1|0|0|0|1|0|1|1
\(#0 = 4, #1 = 4\)
⇒ Unbalanced!
```

- Research question: do balanced crossover operators give an advantage over one-point crossover in optimization problems where the solutions must be balanced?

**Solution Encodings**

- **Direct Binary Coding**: Binary string of length \(N\) such that the number of ones is \(N/2\)

```plaintext
1|0|0|0|1|0|1|1
\(\chi\) point
```

- **Map of Ones Coding**: Integer vector specifying the positions of the \(N/2\) ones in the binary string

```plaintext
1|5|7|8
```

- **Zero Lengths Coding**: Integer vector specifying the run lengths of zeros between consecutive ones

```plaintext
0|3|1|0|0
```

**Counter-Based Crossover**

**Idea**: uniform crossover on the binary coding, using counters to keep track of the multiplicities of zeros and ones

- **Map of Ones Crossover**

**Idea**: uniform crossover on the maps of ones, avoiding the insertion of duplicate positions in the child

```plaintext
p1: 2|4|6|7
\(\chi\)

p2: c
2|5|7|8
```

- **Zero Lengths Crossover**

**Idea**: uniform crossover on the zero-lengths vectors, using an accumulator to track the sums of the run lengths

```plaintext
p1: 1|1|1|0|1
\(\chi\)

p2: c
0|1|1|0|2
```

**Experimental Setting**

- **Optimization Problem**: maximize nonlinearity of balanced Boolean functions \(f : \{0, 1\}^n \rightarrow \{0, 1\}\) for \(n = 6, 7, 8\) variables
- **Experimental Approach**: pairwise comparison of one-point crossover and the three balanced operators with Mann-Whitney-Wilcoxon test (\(\alpha = 0.01\))

**GA Parameters**

- **Breeding Policy**: Steady-state GA with tournament selection
- **Mutation operator**: bit-flip for one-point crossover, swap mutation for balanced operators with \(p_{\text{mut}} = 0.2\)
- **Other Parameters**: population size 50, number of fitness evaluations 500,000

**Results**

- Distribution of nonlinearity values given by the best solution over \(R = 50\) experimental runs:

- One-point crossover is the worst performer among the four operators, especially when the problem size increases
- Map of Ones crossover achieves the best performance over this problem, but its advantage with respect to the other balanced operators is not statistically significant for \(n = 8\) variables