Does Constraining the Search Space of GA Always Help? The Case of Balanced Crossover Operators

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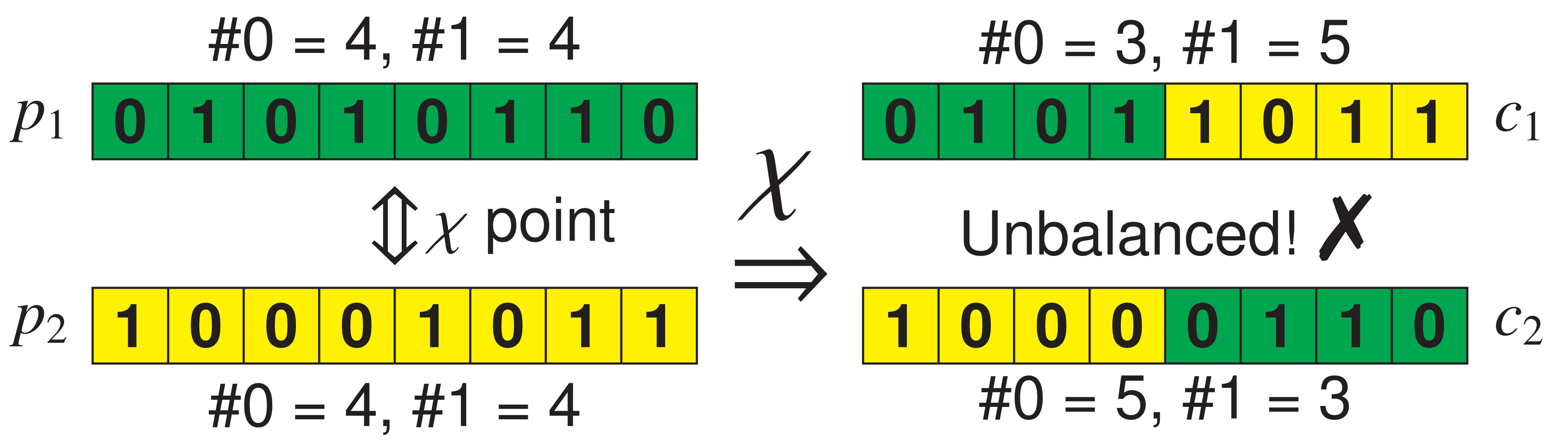
Problem

- In several combinatorial optimization problems, the feasible solutions are represented by *balanced* binary strings, composed of an equal number of zeros and ones
- Classic crossover operators in GA (e.g. one-point) do not preserve balancedness in general:

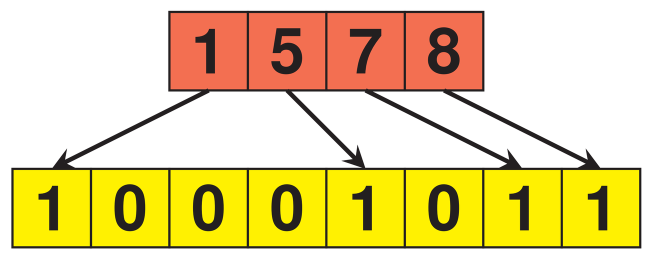


- **Research question:** do *balanced crossover operators* give an advantage over one-point crossover in optimization problems where the solutions must be balanced?

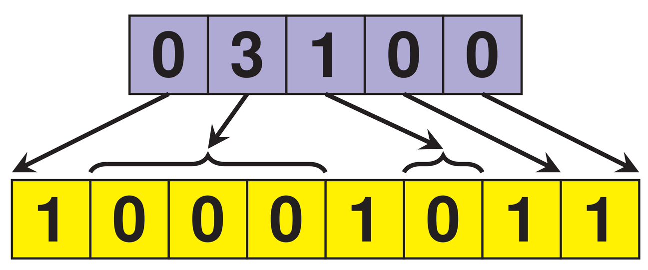
Solution Encodings

- **Direct Binary Coding:** Binary string of length N such that the number of ones is $N/2$

- **Map of Ones Coding:** Integer vector specifying the *positions of the $N/2$ ones* in the binary string

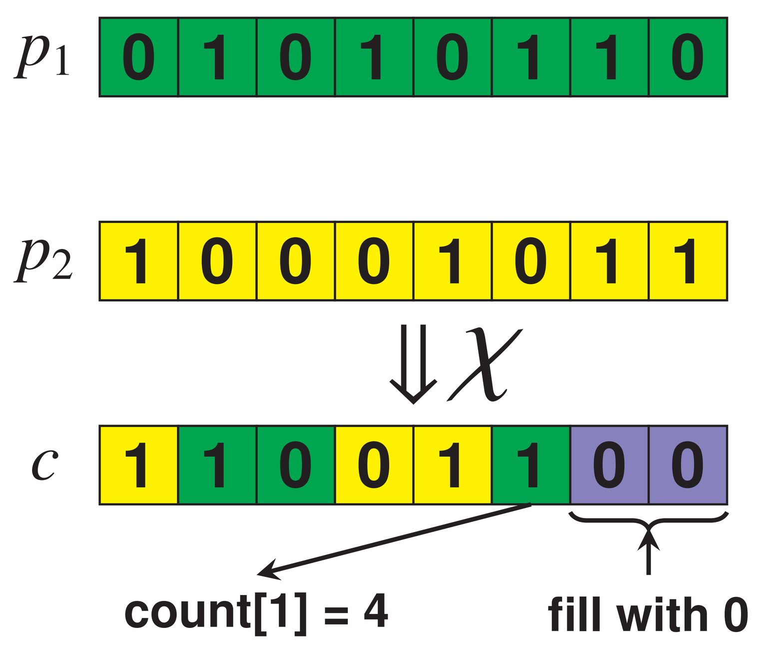


- **Zero Lengths Coding:** Integer vector specifying the *run lengths of zeros* between consecutive ones



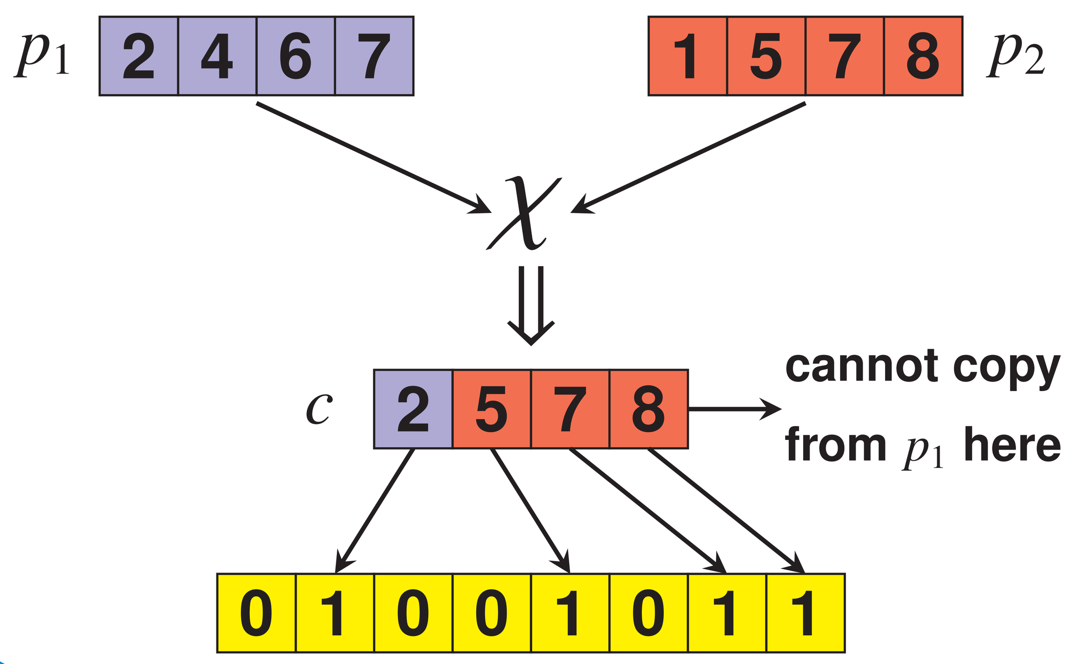
Counter-Based Crossover

Idea: uniform crossover on the binary coding, using *counters* to keep track of the multiplicities of zeros and ones



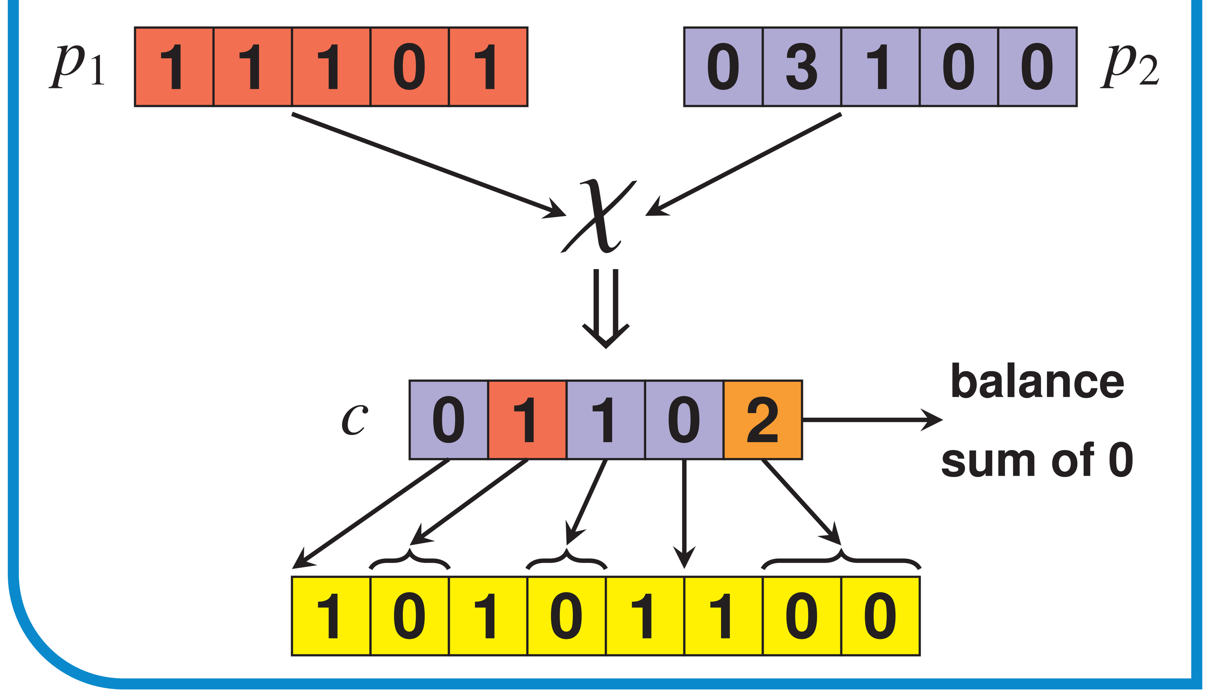
Map of Ones Crossover

Idea: uniform crossover on the maps of ones, avoiding the insertion of duplicate positions in the child



Zero Lengths Crossover

Idea: uniform crossover on the zero-lengths vectors, using an *accumulator* to track the sums of the run lengths



Experimental Setting

- **Optimization Problem:** maximize *nonlinearity* of *balanced Boolean functions* $f : \{0,1\}^n \to \{0,1\}$ for $n = 6, 7, 8$ variables
- **Experimental Approach:** pairwise comparison of one-point crossover and the three balanced operators with *Mann-Whitney-Wilcoxon test* ($\alpha = 0.01$)

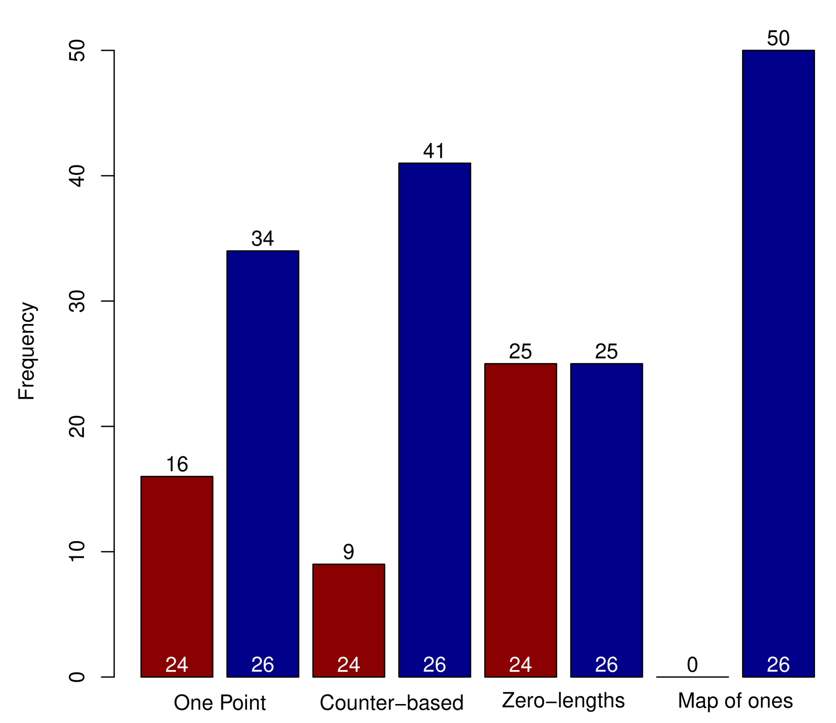
GA Parameters

- **Breeding Policy**: Steady-state GA with tournament selection
- **Mutation operator**: bit-flip for one-point crossover, swap mutation for balanced operators with $p_{mut} = 0.2$
- **Other Parameters**: population size 50, number of fitness evaluations 500 000

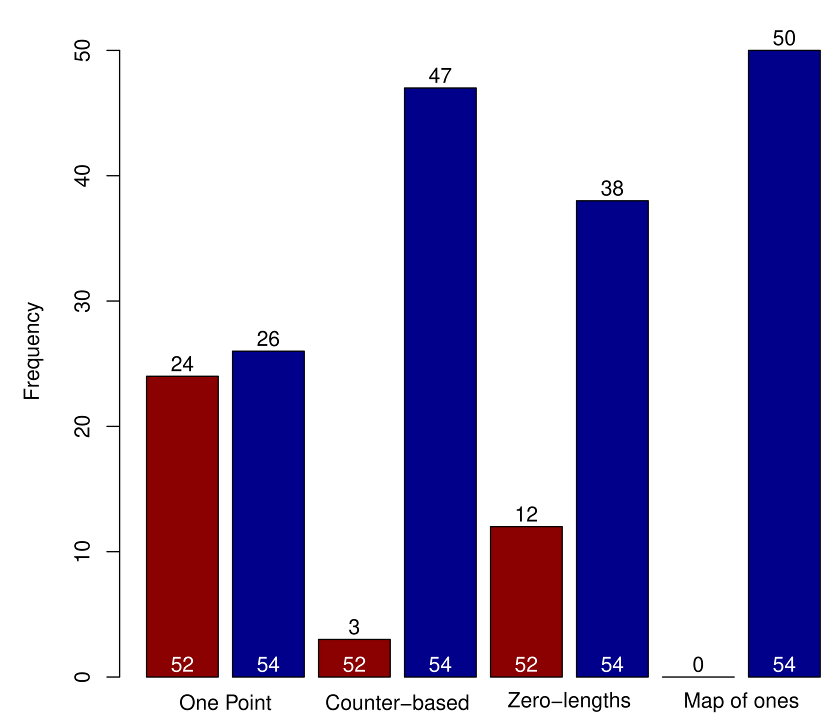
Results

- Distribution of nonlinearity values given by the best solution over $R = 50$ experimental runs:

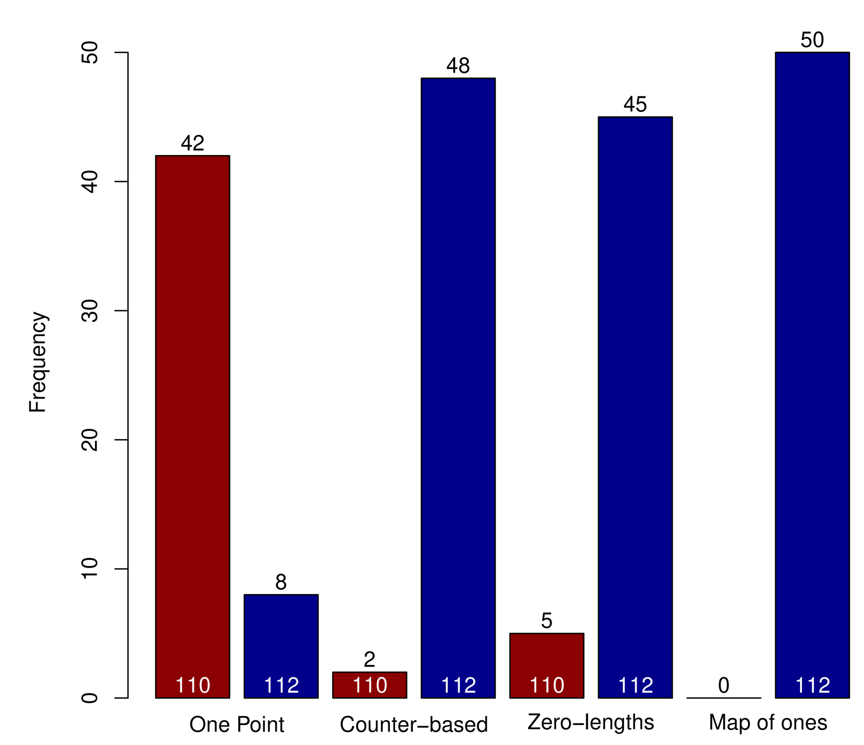
n=6 instance



n=7 instance



n=8 instance



- One-point crossover is the worst performer among the four operators, especially when the problem size increases
- Map of Ones crossover achieves the best performance over this problem, but its advantage with respect to the other balanced operators is not statistically significant for $n = 8$ variables