Asynchrony Immune CA

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Asynchronous CA Model

- One-dimensional array of n binary cells $c = (c_1, \cdots, c_n)$
- Local update rule $f : \{0,1\}^{2r+1} \rightarrow \{0,1\}$
- *Asynchrony mask* $s \in \{0,1\}^{n-2r}$, $s = (s_r, \cdots, s_{n-r})$

$$c_i(t+1) = \begin{cases} f(c_{i-r}, \cdots, c_i, \cdots, c_{i+r}) & \text{, if } s_i = 0 \\ c_i & \text{, if } s_i = 1 \end{cases}$$

Example: $f(c_{i-1}, c_i, c_{i+1}) = c_{i-1} \oplus c_i \oplus c_{i+1}$ (Rule 150)

(a) Synchronous update

$c(t) =$ | **0** | **1** | **1** | **0** | **0** | **1** | **0** | **1** |

⇓

$c(t+1) =$ | **0** | **0** | **1** | **1** | **1** | **0** |

$s =$ | **0** | **0** | **0** | **0** | **0** | **0** |

(b) Asynchronous update

$c(t) =$ | **0** | **1** | **1** | **0** | **0** | **1** | **0** | **1** |

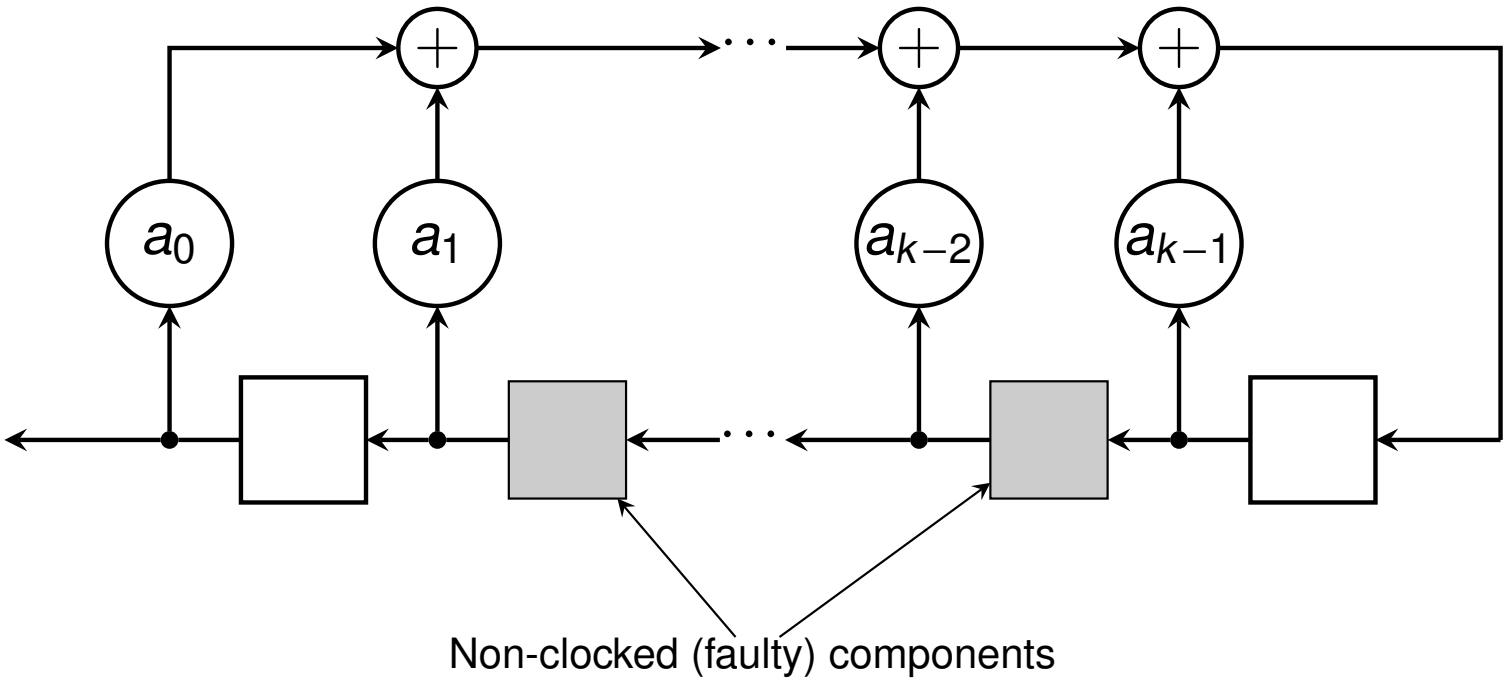
⇓

$c(t+1) =$ | **0** | **1** | **1** | **0** | **1** | **0** |

$s =$ | **0** | **1** | **0** | **1** | **0** | **0** |

Side-Channel Attacks

- Target the implementation of a cipher
- Idea: Gain information on the internal state of a cipher by injecting faults and tracking them
- Considered attack: clock faults



Towards Asynchrony Immunity... (Problem Statement)

Definition (Balancedness)

A CA $F : \{0,1\}^n \to \{0,1\}^{n-2r}$ is balanced if every configuration of $n-2r$ cells has exactly 2^{2r} preimages under F

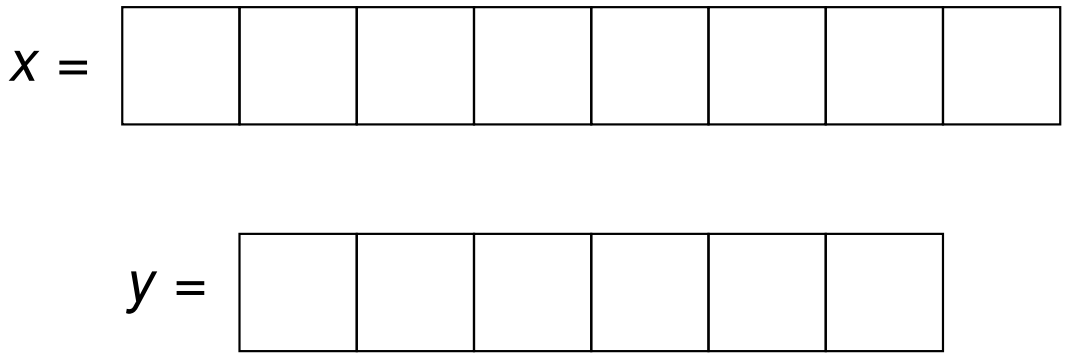
Remark: Balancedness is a critical cryptographic property: unbalanced CA have statistical biases exploitable by an attacker

Problem

Which CA stay balanced under clock faults?

Towards Asynchrony Immunity... (Game Formulation)

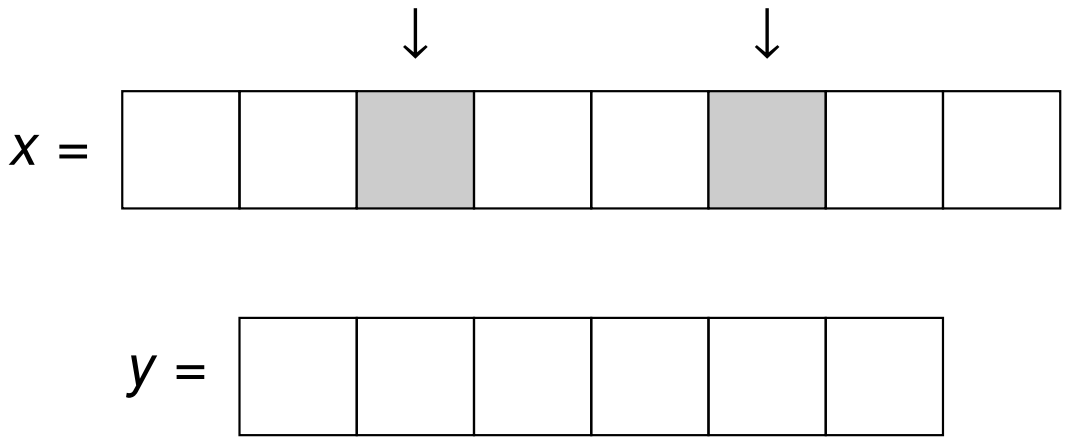
1. The *user* chooses a balanced CA $F : \{0,1\}^n \to \{0,1\}^{n-2r}$



Example: CA $F : \{0,1\}^8 \to \{0,1\}^6$ induced by rule 60,
$f(x_1, x_2, x_3) = x_2 \oplus x_3$

Towards Asynchrony Immunity... (Game Formulation)

2. The *adversary* blocks the update of $j < t$ cells



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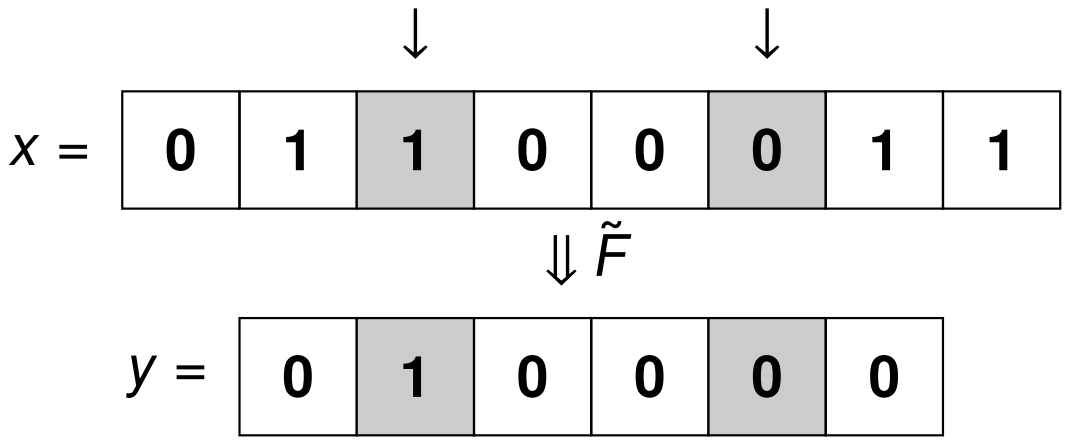
Towards Asynchrony Immunity... (Game Formulation)

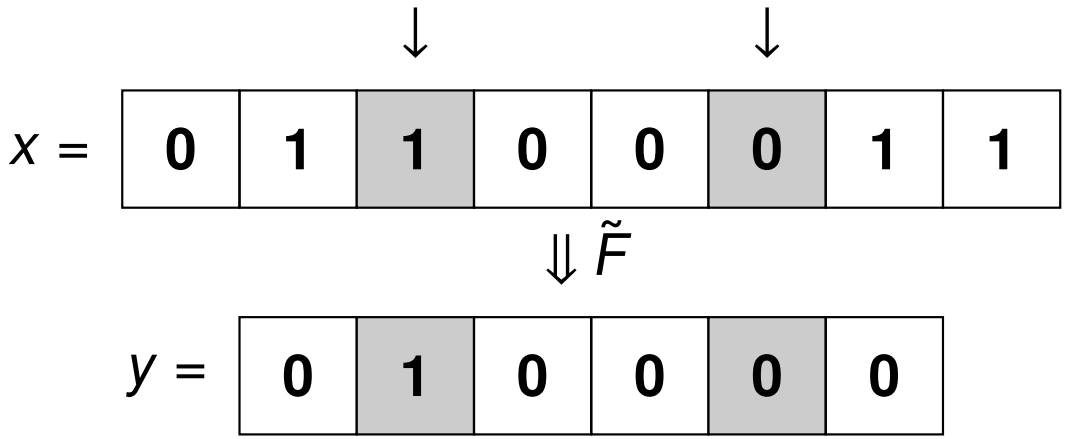
3. The user evaluates the global rule of the asynchronous CA $\tilde{F}$



Example: CA $F : \{0,1\}^8 \to \{0,1\}^6$ induced by rule 60,
$f(x_1, x_2, x_3) = x_2 \oplus x_3$

Towards Asynchrony Immunity... (Game Formulation)

- Outcome: if $\tilde{F}$ is still balanced, then the user wins. Otherwise, the adversary wins



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Asynchrony Immune CA $\Leftrightarrow$ winning strategies for the player

Asynchrony Immunity: Definition

Difference with Resiliency Game [Chor85]: choice of the *values* of the cells by the adversary $\Rightarrow$ stronger attack hypothesis

Definition

A local rule $f : \{0,1\}^{2r+1} \to \{0,1\}$ is (t,n)-asynchrony immune ((t,n)-AI) if, for all $0 < k \leq n-2r$ and for all asynchrony masks s of weight at most t, the synchronous CA $F : \{0,1\}^{k+2r} \to \{0,1\}^k$ and the asynchronous CA $\tilde{F} : \{0,1\}^{k+2r} \to \{0,1\}^k$ are balanced

Asynchrony Immunity: Basic Properties

- Idea: search for AI rules with additional crypto properties (high nonlinearity), pruning the search space through symmetries
- Reflection: $f^R(x_1, \cdots, x_{2r+1}) = f(x_{2r+1}, \cdots, x_1)$
- Complement: $f^C(x_1, \cdots, x_{2r+1}) = 1 \oplus f(x_1, \cdots, x_{2r+1})$

Lemma

Let $f : \{0,1\}^{2r+1} \to \{0,1\}$ be a (t,n)-AI local rule. Then, f^R and f^C are (t,n)-AI as well

Search for AI Rules

- Goal: find nonlinear $(3,10)$-AI rules
- In elementary CA, only rule 60 is $(3,10)$-AI, but it is linear
- Extended the search to the 2^{2^4} 4-neighborhood local rules
- 18 $(3,10)$-AI rules, 12 of which nonlinear
- All the rules found are *center-permutive*, i.e.

$$f(x_{i-r},\cdots,x_i,\cdots,x_{i+r}) = x_i \oplus g(x_{i-r},\cdots,x_{i+r})$$

Conjecture

Let f be a (t,n)-asynchrony immune local rule. Then, f is center-permutive

Conclusions

Summary:

- Introduction of a new cryptographic property modelled using asynchronous CA
- Possible applications in side-channel countermeasures for CA-based ciphers
- Performed a computer search of $(3, 10)$-AI nonlinear 4-neighborhood rules

Perspectives:

- Prove (or disprove) the center-permutivity conjecture
- Restrict the search to surjective CA [Amoroso72]
- Design a family of bent (maximally nonlinear) AI rules
- Cryptanalyze existing ciphers (e.g., KECCAK [Keccak11])

References

- [Amoroso72] Amoroso, S., Patt, Y.N.: Decision Procedures for Surjectivity and Injectivity of Parallel Maps for Tessellation Structures. J. Comput. Syst. Sci. 6(5): 448-464 (1972)
- [Chor85] B. Chor, O. Goldreich, J. Hastad, J. Freidmann, S. Rudich, R. Smolensky: The bit extraction problem or t-resilient functions. In: Proceedings of FOCS '85, pp. 396–407. IEEE Computer Society (1985)
- [Keccak11] Bertoni, G., Daemen, J., Peeters, M., Assche, G.V.: The Keccak reference. `http://keccak.noekeon.org/` (2011)