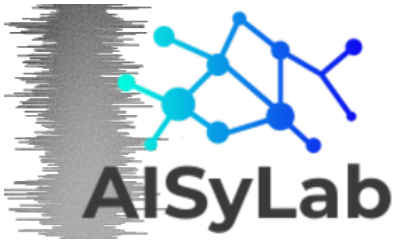


Radboud University

Building Correlation Immune Functions from Sets of Mutually Orthogonal CA

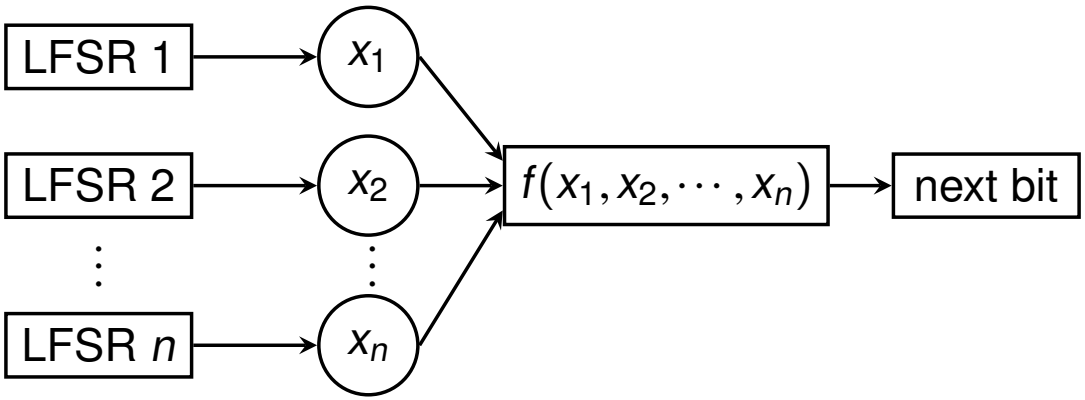
Luca Mariot, Luca Manzoni

luca.mariot@ru.nl

AUTOMATA 2022 – 11 October 2022

Correlation Immune Boolean Functions in Crypto

- **Boolean function**: mapping $f : \{0,1\}^n \rightarrow \{0,1\}$
- **Correlation Immunity of order t**: output of f is statistically independent from any subset of at most t inputs



Applications in symmetric crypto:

- Combine the output of n LFSR for **stream encryption** [C21]
- CA-based **pseudorandom number generators** [L13, F14]
- Masking countermeasures for **side-channel analysis** [C12, K14]

Representation of Boolean Functions

- **Truth table**: vector Ω_f specifying $f(x)$ for all $x \in \mathbb{F}_2$
- **Walsh Transform**: correlation with the *linear* functions defined as $\omega \cdot x = \omega_1 x_1 \oplus \cdots \oplus \omega_n x_n$

$$\hat{F}(\omega) = \sum_{x \in \mathbb{F}_2^n} (-1)^{f(x) \oplus \omega \cdot x}$$

(x_1, x_2, x_3)	000	100	010	110	001	101	011	111
Ω_f	0	1	1	0	1	0	0	1
$\hat{F}(\omega)$	0	0	0	0	0	0	0	8

Example: $n = 3$, $f(x_1, x_2, x_3) = x_1 \oplus x_2 \oplus x_3$

Correlation Immunity: Walsh Characterization

- f is t-correlation immune iff $W_f(a) = 0$ for all a s.t. $1 \leq HW(a) \leq t$, where HW is the Hamming weight of a [X88]

Example: $t = 2$

(x_1, x_2, x_3)	000	**100**	**010**	**110**	**001**	**101**	**011**	111
Ω_f	0	1	1	0	1	0	0	1
$\hat{F}(\omega)$	0	0	0	0	0	0	0	8

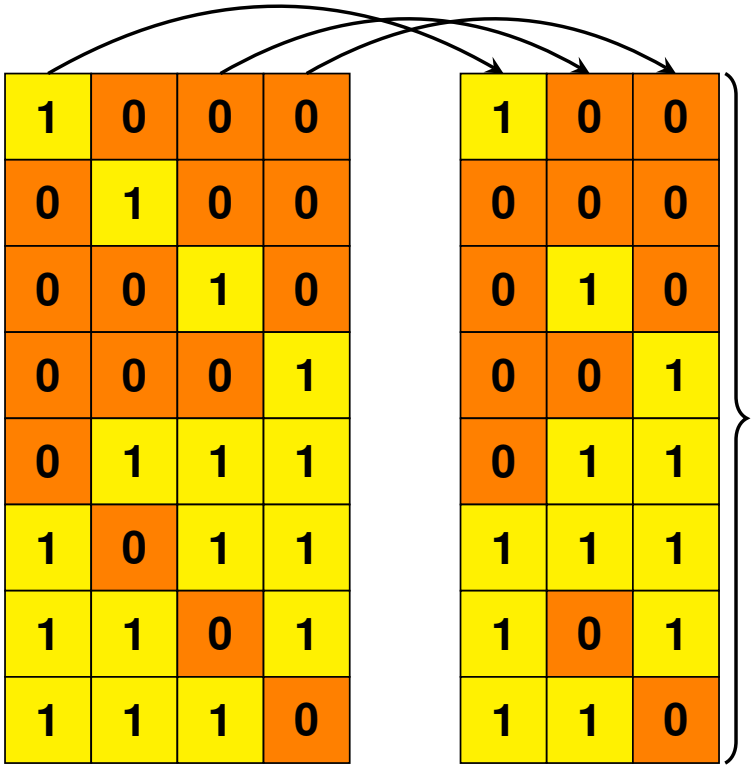
$\Downarrow$

f is 2-order correlation immune

- Relevance in side-channel: t-order CI functions $\Rightarrow$ Boolean masking resistant to SCA attacks of order t

Orthogonal Arrays (OA)

- (N,k,s,t) **Orthogonal Array**: $N \times k$ matrix A such that each t-uple occurs $\lambda = N/s^t$ times in each $N \times t$ submatrix.



Example: OA $(8,4,2,3)$

$\Rightarrow$ Each 3-bit vector $(x_1,x_2,x_3) \in \{0,1\}^3$ appears once in the submatrix with columns 1, 3, 4

- Applications in statistics, coding theory, cryptography

Correlation Immunity: OA Characterization

- **Support** of f: sets of input vectors x that map to 1 under f

Truth table

x_1	x_2	x_3	$f(x)$
0	0	0	0
0	0	1	**1**
0	1	0	**1**
0	1	1	0
1	0	0	**1**
1	0	1	0
1	1	0	0
1	1	1	**1**

Support

x_1	x_2	x_3
0	0	1
0	1	0
1	0	0
1	1	1

$\Downarrow$

$OA(4,3,2,2)$

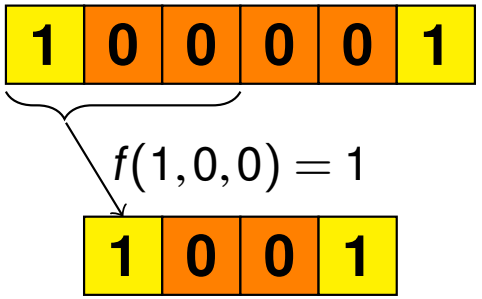
Theorem ([C92])

$f : \{0,1\}^n \to \{0,1\}$ is t-order CI $\Leftrightarrow$ Support of f is an $OA(N,n,2,t)$, with $N = |Supp(f)|$

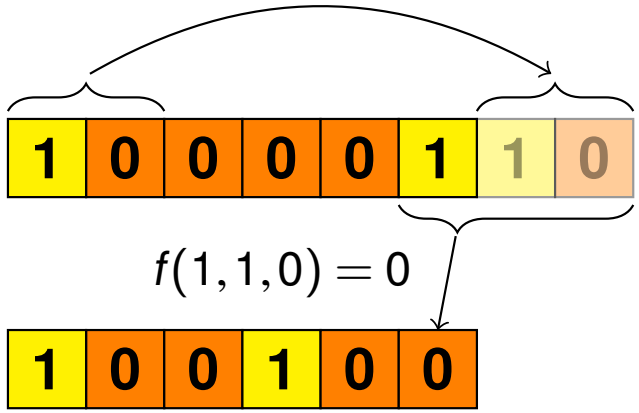
Cellular Automata

- One-dimensional Cellular Automaton (CA): a discrete parallel computation model composed of a finite array of n cells

Example: $n = 6$, $d = 3$, $f(s_i, s_{i+1}, s_{i+2}) = s_i \oplus s_{i+1} \oplus s_{i+2}$ (rule 150)



No Boundary CA – NBCA



Periodic Boundary CA – PBCA

- Each cell updates its state $s \in \{0,1\}$ by evaluating a local rule $f : \{0,1\}^d \to \{0,1\}$ on itself and the $d-1$ cells on its right

Mutually Orthogonal Latin Squares (MOLS)

Definition

A *Latin square* is a $n \times n$ matrix where all rows and columns are permutations of $[n] = \{1, \cdots, n\}$. Two Latin squares are *orthogonal* if their superposition yields all the pairs $(x, y) \in [n] \times [n]$.

1	3	4	2
4	2	1	3
2	4	3	1
3	1	2	4

1	4	2	3
3	2	4	1
4	1	3	2
2	3	1	4

1,1	3,4	4,2	2,3
4,3	2,2	1,4	3,1
2,4	4,1	3,3	1,2
3,2	1,3	2,1	4,4

- k-**MOLS**: set of k pairwise orthogonal Latin squares
- k-MOLS are equivalent to $OA(n^2, k, n, 2)$

Latin Squares through Bipermutive CA (1/2)

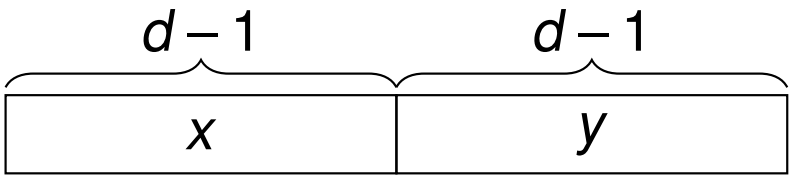
- Bipermutive CA: denoting $\mathbb{F}_2 = \{0,1\}$, local rule f is defined as

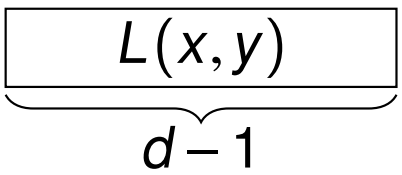
$$f(x_1, \cdots, x_d) = x_1 \oplus \varphi(x_2, \cdots, x_{d-1}) \oplus x_d$$

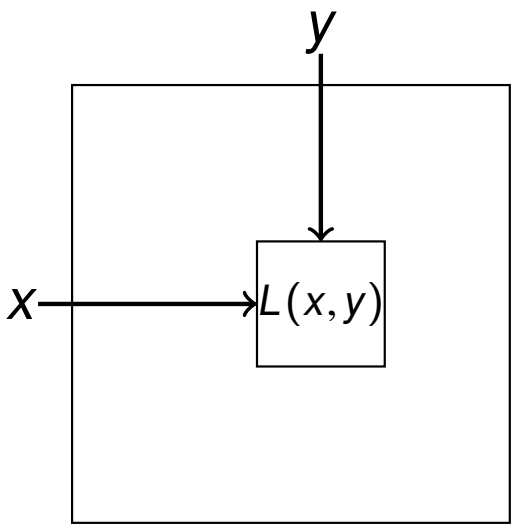
- $\varphi : \mathbb{F}_2^{d-2} \to \mathbb{F}_2$: generating function of f

Lemma ([E93, M16])

A CA $F : \mathbb{F}_2^{2(d-1)} \to \mathbb{F}_2^d$ with bipermutive rule $f : \mathbb{F}_2^d \to \mathbb{F}_2$ generates a Latin square of order $N = 2^{d-1}$

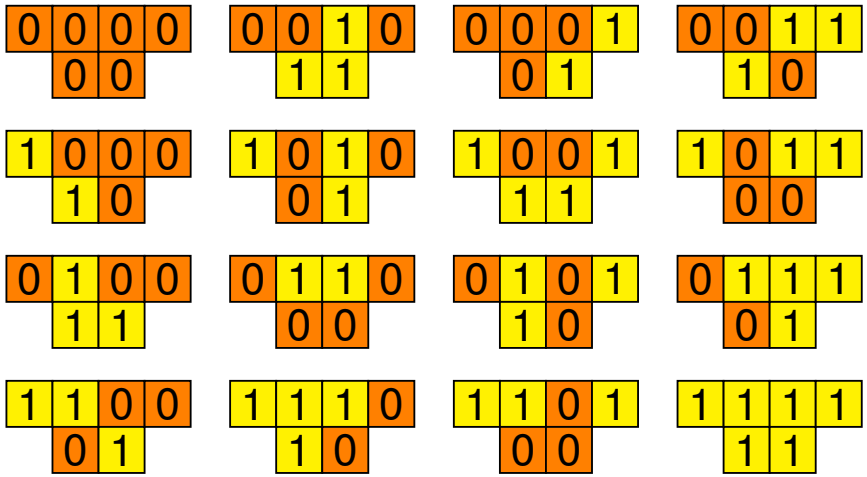






Latin Squares through Bipermutive CA (2/2)

- Example: CA $F : \mathbb{F}_2^4 \to \mathbb{F}_2^2$, $f(x_1, x_2, x_3) = x_1 \oplus x_2 \oplus x_3$ (Rule 150)
- Encoding: $00 \mapsto 1, 10 \mapsto 2, 01 \mapsto 3, 11 \mapsto 4$



(a) Rule 150 on 4 bits

1	4	3	2
2	3	4	1
4	1	2	3
3	2	1	4

(b) Latin square L_{150}

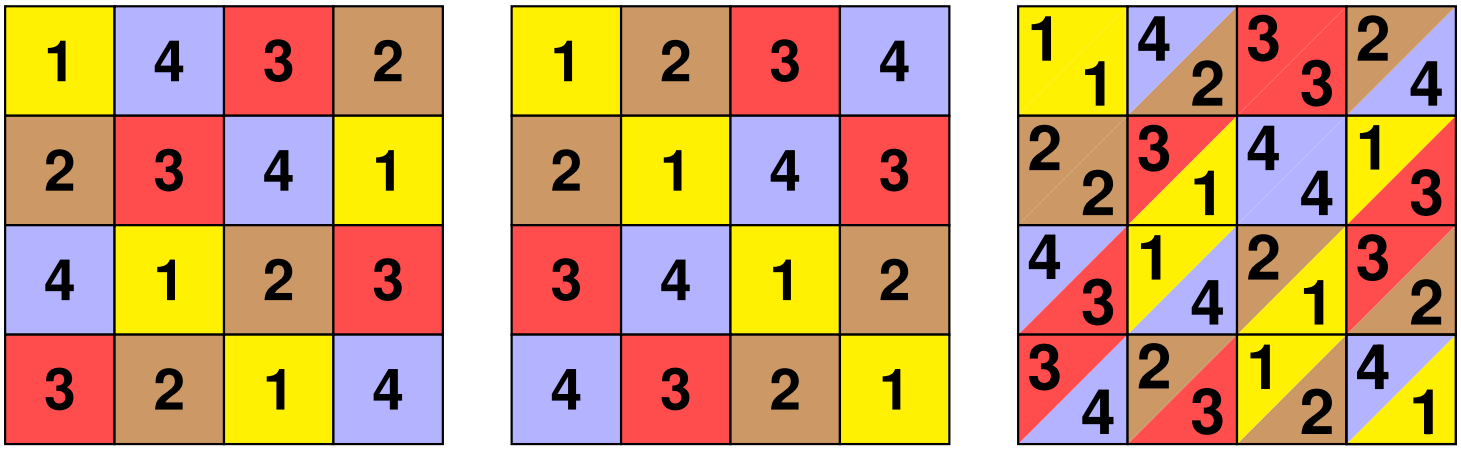
k-Mutually Orthogonal Cellular Automata (MOCA): k bipermutive CA F, G generating a set of k-MOLS

Example with Linear CA: Rules 90-150

- Bipermutive Linear rule: $f(x) = x_1 \oplus a_2 x_2 \oplus \cdots \oplus a_{d-1} x_{d-1} \oplus x_d$
- Polynomial rule: $P_f(X) = 1 + a_2 X + \cdots + a_{d-1} X^{d-2} + X^{d-1}$

Theorem ([M20])

Two bipermutive linear rules generates OCA if and only if their associated polynomials are coprime



(a) Rule 150 (b) Rule 90 (c) Superposition

Figure: $P_{150}(X) = 1 + X + X^2$, $P_{90}(X) = 1 + X^2$ (coprime)

Construction of CI functions from MOCA

Procedure:

- **Input**: k-MOCA $F_1, \cdots, F_k : \mathbb{F}_2^{2b} \to \mathbb{F}_2^b$ of diameter $d = b + 1$
- Construct the set of k-MOLS over $[2^b]$ using the combinatorial algorithms from [M17, M18]
- "Linearize" the k-MOLS in a $2^b \times k$ OA
- Convert each entry in the OA in **binary**
- **Output**: the converted binary array

Rule 150

1	4	3	2
2	3	4	1
4	1	2	3
3	2	1	4

Rule 90

1	2	3	4
2	1	4	3
3	4	1	2
4	3	2	1

$A =$

$L_{90}L_{150}$
1 1
2 4
3 3
4 2
2 2
1 3
4 4
3 1
3 4
4 1
1 2
2 3
4 3
3 2
2 1
1 4

Lemma

The output array is an OA$(2^b, kb, 2, 2)$.

Computational Search Results

- **Consequence**: k-MOCA generate supports of Boolean functions with $n = kb$ variables with CI order at least 2
- Exhaustive search of 3-MOCA with $d = 4, 5$, $d = b + 1$
- Checked CI order with Walsh Transform

Table: Classification of correlation immune functions generated by 3-MOCA of diameter $d \in \{4, 5\}$.

d	#3-MOCA	n	w_H	CI	$\#CI$	Minw_H
4	2	9	64	3	2	20
5	36	12	256	3	27	24
5	36	12	256	4	9	24

- **Main finding**: all functions are at least 3-CI

Conclusions and Future Work

Wrapping up:

- We proved that k-MOCA generate correlation immune functions of order at least 2
- Experimentally, we noticed that k-MOCA functions actually have order at least 3

Future directions:

- Theoretically: are there MOCA that give CI functions with $t = 2$?
- Practically: reduce the Hamming weight of the functions, using evolutionary algorithms [M21]

References

- [C92] P. Camion, C. Carlet, P. Charpin, N. Sendrier: On Correlation-Immune Functions. Proceedings of CRYPTO 1991, pp. 86-100 (1992)
- [C21] C. Carlet: Boolean functions for cryptography and coding theory. Cambridge University Press (2021)
- [C12] C. Carlet, J.-L. Danger, S. Guilley, H. Maghrebi: Leakage Squeezing of Order Two. Proceedings of INDOCRYPT 2012, pp. 120-139 (2012)
- [E93] Eloranta, K.: Partially Permutive Cellular Automata. Nonlinearity 6(6), 1009–1023 (1993)
- [F14] E. Formenti, K. Imai, B. Martin, J.-B. Yunès: Advances on Random Sequence Generation by Uniform Cellular Automata. Computing with New Resources 2014: 56-70 (2014)
- [K14] S. Karmakar, D.R. Chowdhury: Leakage squeezing using cellular automata and its application to scan attack. J. Cell. Autom. 9(5-6) (2014) 417–436
- [L13] A. Leporati and L. Mariot: 1-Resiliency of Bipermutive Cellular Automata Rules. Proceedings of Automata 2013, pp. 110-123 (2013)
- [M21] L. Mariot: Deriving Smaller Orthogonal Arrays from Bigger Ones with Genetic Algorithm. CoRR abs/2111.13047 (2021)
- [M20] L. Mariot, M. Gadouleau, E. Formenti, A. Leporati: Mutually orthogonal latin squares based on cellular automata. Des. Codes Cryptogr. 88(2): 391-411 (2020)
- [M18] L. Mariot, A. Leporati: Inversion of Mutually Orthogonal Cellular Automata. Proceedings of ACRI 2018, pp. 364-376 (2018)
- [M17] L. Mariot, E. Formenti, A. Leporati: Enumerating Orthogonal Latin Squares Generated by Bipermutive Cellular Automata. Proceedings of AUTOMATA 2017, pp. 151-164 (2017)
- [M16] L. Mariot, E. Formenti, A. Leporati: Constructing Orthogonal Latin Squares from Linear Cellular Automata. In: Exploratory papers of AUTOMATA 2016 (2016)
- [X88] G.-Z. Xiao, J. L. Massey: A spectral characterization of correlation-immune combining functions. IEEE Trans. Inf. Theory 34(3): 569-571 (1988)