Tip the Balance: Partially Unbalanced Crossover by Adaptive Bias

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This work is a follow-up of the following paper published in Swarm and Evolutionary Computation:


which is an extended version of a short paper presented at GECCO 2019 [MMT19]
Classic crossover and balancedness

In some optimization problems, feasible solutions are represented by *balanced* binary strings, composed of an equal number of zeros and ones.

In general, classic GA crossover operators in GA do not preserve balancedness.

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Counter-based crossover

- Uniform crossover with *counters* to keep track of the multiplicities of zeros and ones [MCD98]
- copy the other value when the threshold is reached

\[
P_1: 010101110 \quad \chi \Rightarrow \quad \begin{array}{cccccccc}
0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\
\end{array}
\]

\[
P_2: 100010111
\]

- Gives an advantage over one-point crossover, but finds less optimal solutions as the problem size grows [MMT20]
Tip the balance: Slightly enlarge the search space by allowing some unbalancedness in the offspring.

Keep copying the wrong value with probability $p$, and switch to the correct one with probability $1 - p$.
Adaptive bias mechanism

- **Adaptive bias**: probability $p$ is updated with a *geometric cooling mechanism* similar to simulated annealing [KGV83]
  \[ p \leftarrow \alpha \cdot p \text{, where } \alpha \in (0, 1) \]

- **Weighted penalty factor** added to the fitness function:
  \[ w_{pen}(x) = (1 - p) \cdot |w_H(x) - k| \text{,} \]
  where $w_H(x)=$number of 1s in $x$, and $k$ is the target weight
Experimental validation

- **Problem**: balanced nonlinear Boolean functions [C21]
- The truth table of a Boolean function $f : \{0, 1\}^n \rightarrow \{0, 1\}$ of $n$ variables is encoded by a $2^n$-bit string
- **Balancedness constraint**: the truth table must be composed of an equal number of 0s and 1s
- **Optimization goal**: maximize the nonlinearity $Nl(f)$ of $f$

$$fit(f) = Nl(f) - wpen(f) = Nl(f) - (1 - p) \cdot |2^{n-1} - w_H(f)|$$

- Same experimental setting used in [MMT20] to compare with counter-based crossover and map-of-ones
Results

Distribution of fitness values over 50 experimental runs for Boolean functions of $n = 7$ variables.

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Conclusions and Future Research

Conclusions:

▶ The partially unbalanced crossover generated slightly more optimal solutions than other crossover methods
▶ However, there are no statistically significant differences in the best fitness distributions

Future Directions:

▶ Better analyze and tune the adaptive bias parameters to boost performances, as well as other GA parameters
▶ Apply the adaptive bias strategy to other problems, e.g. orthogonal arrays [MPJL18], orthogonal Latin squares [MPJL17], disjunct matrices [KPMJL18], plateaued functions [ML15], bent functions [PJ16, PKMJL18] ...
References


