Tip the Balance: Partially Unbalanced Crossover by Adaptive Bias

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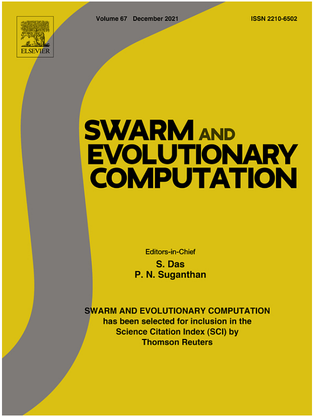
Broader reference

This work is a follow-up of the following paper published in Swarm and Evolutionary Computation:

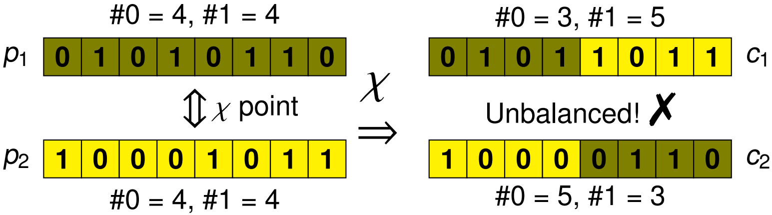
L. Manzoni, L. Mariot, E. Tuba: *Balanced crossover operators in Genetic Algorithms.* Swarm Evol. Comput. 54: 100646 (2020)



which is an extended version of a short paper presented at GECCO 2019 [MMT19]

Classic crossover and balancedness

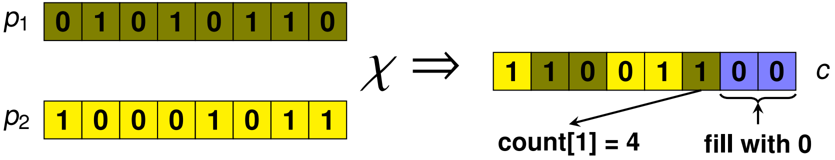
- In some optimization problems, feasible solutions are represented by *balanced* binary strings, composed of an equal number of zeros and ones



- In general, classic GA crossover operators in GA do not preserve balancedness

Counter-based crossover

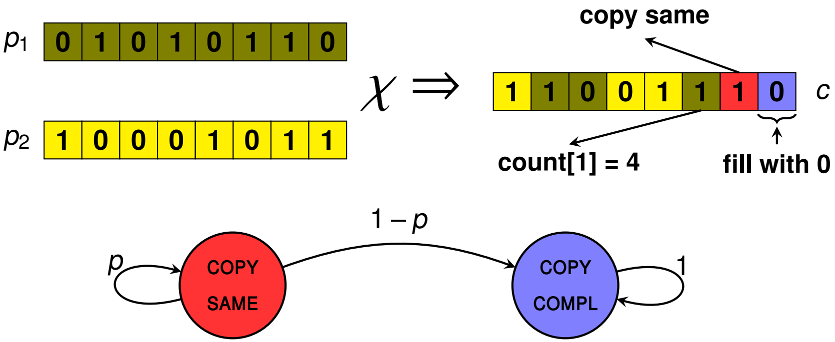
- Uniform crossover with *counters* to keep track of the multiplicities of zeros and ones [MCD98]
- copy the other value when the threshold is reached



- Gives an advantage over one-point crossover, but finds less optimal solutions as the problem size grows [MMT20]

Partially unbalanced crossover

- **Tip the balance:** Slightly enlarge the search space by allowing some unbalancedness in the offspring
- Keep copying the wrong value with probability p, and switch to the correct one with probability $1 - p$



Adaptive bias mechanism

- **Adaptive bias**: probability p is updated with a *geometric cooling mechanism* similar to simulated annealing [KGV83]

$$p \leftarrow \alpha \cdot p \ , \ \ where \ \ \alpha \in (0,1)$$

- **Weighted penalty factor** added to the fitness function:

$$w_{pen}(x) = (1-p) \cdot |w_H(x) - k| \ ,$$

where $w_H(x)$=number of 1s in x, and k is the target weight

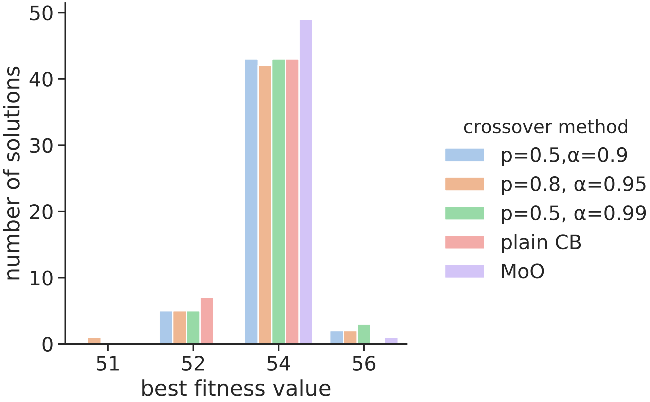
Experimental validation

- **Problem**: balanced nonlinear Boolean functions [C21]
- The truth table of a Boolean function $f : \{0,1\}^n \to \{0,1\}$ of n variables is encoded by a 2^n-bit string
- **Balancedness constraint**: the truth table must be composed of an equal number o 0s and 1s
- **Optimization goal**: maximize the *nonlinearity* $Nl(f)$ of f

$$fit(f) = Nl(f) - wpen(f) = Nl(f) - (1-p) \cdot |2^{n-1} - w_H(f)|$$

- Same experimental setting used in [MMT20] to compare with counter-based crossover and map-of-ones

Results



Distribution of fitness values over 50 experimental runs for Boolean functions of $n = 7$ variables

Conclusions and Future Research

Conclusions:

- The partially unbalanced crossover generated slightly more optimal solutions than other crossover methods
- However, there are no statistically significant differences in the best fitness distributions

Future Directions:

- Better analyze and tune the adaptive bias parameters to boost performances, as well as other GA parameters
- Apply the adaptive bias strategy to other problems, e.g. orthogonal arrays [MPJL18], orthogonal Latin squares [MPJL17], disjunct matrices [KPMJL18], plateaued functions [ML15], bent functions [PJ16, PKMJL18] ...

References

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